Readers' Forum

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the AIAA Journal *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Rise of Total Pressure in Frictional Flow"

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SOME additional remarks are made concerning the subject that was discussed in Ref. 1, in which it was stated that, contrary to common assumptions, the total pressure can show an increase in frictional flow, especially near stagnation areas. A topic that is directly related to this and has practical significance, but was not mentioned, is the viscous effect on pitot tube readings when determining the total pressure of the flow,[2,3] which is not uncommon to many experimentalists. With this specific application in mind, some of the results of the preceding article may be considered in more detail.

Using the same notation as that employed by Issa,[1] one can write the momentum equation in an incompressible flow as

$$\frac{\partial p}{\partial x_i} + u_j \frac{\partial \rho u_i}{\partial x_j} = \frac{\partial \sigma_{ij}}{\partial x_j} = \mu \frac{\partial^2 u_i}{\partial x_j \partial x_j} \tag{1}$$

where the stress term has been evaluated for laminar Newtonian flow. Let us consider the case of stagnation flow (either the plane two-dimensional case considered by Issa or the three-dimensional axisymmetrical case, for both of which analytic solutions exist[4,5]), for which the stagnation streamline coincides with the y axis. Evaluating the momentum equation along this streamline, indicated by the subscript s, yields for the two-dimensional stagnation flow the following expression, putting $x_2 = y$ and $u_2(0, y, 0) = v_s(y)$:

$$\frac{\mathrm{d}p_s}{\mathrm{d}y_s} + \rho v_s \frac{\mathrm{d}v_s}{\mathrm{d}y_s} = \frac{\mathrm{d}p_{ts}}{\mathrm{d}y_s} = \left(\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2}\right)_s = \mu \frac{\mathrm{d}^2 v_s}{\mathrm{d}y_s^2} \tag{2}$$

which corresponds to the result of Eq. (9) of the original paper. Though the final expression suggests that the change of the total pressure can be attributed solely to the action of the normal stress σ_{22}, evaluation of the individual stress terms shows that this is not correct:

$$\begin{aligned} \sigma_{21} &= \mu x f'' & \qquad \frac{\partial \sigma_{21}}{\partial x_1} &= \mu f'' \\ \sigma_{22} &= -2\mu f' & \qquad \frac{\partial \sigma_{22}}{\partial x_2} &= -2\mu f'' \end{aligned} \tag{3}$$

As can be seen, the magnitude of the contribution of the shearing stress is half that of the normal stress, whereas the former works to decrease the total pressure and the latter to increase it. (Note that although f'' is positive, the direction of integration along the streamline is that of negative y.)

The expression for the stagnation pressure p_0 [Eq. (11) of the original paper] is repeated here:

$$p_0 = p_{t\infty} + \mu a \tag{4}$$

where, instead of introducing the rather ambiguous viscous layer thickness δ, it is now more convenient to consider the physical interpretation of a, viz.,

$$a = \frac{\mathrm{d}U_e}{\mathrm{d}x} \tag{5}$$

where U_e is the tangential velocity at the edge of the viscous layer. In comparison to the plane two-dimensional stagnation flow, the axisymmetric stagnation flow yields a slightly different result[4,5]:

$$p_0 = p_{t\infty} + 2\mu a \tag{6}$$

where now

$$a = \frac{\mathrm{d}U_e}{\mathrm{d}r} \tag{7}$$

with $r = \sqrt{(x^2 + z^2)}$ the distance from the symmetry axis.

These results can now be used for the calculation of the viscous effect on the stagnation pressure for, respectively, a circular cylinder and a sphere placed in a parallel flow, for the limit of large Reynolds number.[5] In that case the viscous layer thickness is small with respect to the body radius, and the local stagnation area can be approximated with the corresponding stagnation flow. Taking the inviscid outer flow distribution from potential theory, we find

Cylinder:

$$U_e(x) = 2(U_\infty x/R) \qquad p_0 = p_{t\infty} + 2(\mu U_\infty/R) \tag{8}$$

Sphere:

$$U_e(r) = \tfrac{3}{2}(U_\infty r/R) \qquad p_0 = p_{t\infty} + 3(\mu U_\infty/R) \tag{9}$$

Although by a slightly different argument, Homann[5] arrived at the same results. In addition, he formed an effective radius by adding the displacement thickness to R to extend the Reynolds number range of applicability. This effect can be accommodated in the same way here but is not pursued here any further.

The value of the stagnation pressure coefficient, $c_{p0} = (p_0 - p_\infty)/\frac{1}{2}\rho U_\infty^2$, becomes

Cylinder:

$$c_{p0} = 1 + (8/Re) \tag{10}$$

Sphere:

$$c_{p0} = 1 + (12/Re)$$

where $Re = 2U_\infty R/\nu$ is the Reynolds number based on the cylinder/sphere diameter. For the Stokes flow the viscous effect of the stagnation pressure on a sphere is given by [see Eq. (18) of original paper]

Stokes flow:

$$c_{p0} = 6/Re \tag{11}$$

Note that in this expression, which is valid for the case of small Reynolds numbers, the viscous effect term is of the same functional form in its dependence on Reynolds number, but only half as large as that which is valid for the large Reynolds number limit.

References

[1] Issa, R. I., "Rise of Total Pressure in Frictional Flow," *AIAA Journal*, Vol. 33, No. 4, 1995, pp. 772–774.

[2] Bryer, D. W., and Pankhurst, R. C., *Pressure-Probe Methods for Determining Wind Speed and Flow Direction*, National Physical Lab., Her Majesty's Stationery Office, London, 1971.

[3] Hurd, C. W., Chesky, K. P., and Shapiro, A. H., "Influence of Viscous Effects on Impact Tubes," *Journal of Applied Mechanics*, Vol. 20, June 1953, pp. 253–256.

[4] Schlichting, H., *Boundary Layer Theory*, 7th ed., McGraw–Hill, New York, 1979.

Received June 9, 1995; accepted for publication Aug. 10, 1995.

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[5]Homann, F., "Der Einfluss grosser Zähigkeit bei der Strömung um den Zylinder und um die Kugel," *ZAMM*, Vol. 16, June 1936, pp. 153–164; translated as "The Effect of High Viscosity on the Flow Around a Cylinder and Around a Sphere," NACA TM 1334, June 1952.

Reply by the Author to B. W. van Oudheusden

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THE Comment is very useful in expanding the scope of the original article and in highlighting the important application to the pitot tube device. The original analysis is extended to the case of axisymmetric stagnation flow, and results similar to those obtained for the previously considered plane flow case are reached.

However, the writer of the Comment has misinterpreted some of the statements in the article. Whereas the text states that the positive source causing the rise in total pressure is solely related to the normal stresses, the writer has incorrectly understood this to mean that it is only the normal stresses that cause the change in total pressure. The original article does not imply in any way that there is no negative though smaller contribution due to the shear stresses as indeed there is. It merely states that only the normal stresses in this particular case contribute to the increase in total pressure.

The author agrees that the introduction of the relationship for a in Eq. (5) of the Comment possibly provides a more physical (hence useful) interpretation of the quantity. The original expression [Eq. (13) of the paper] was only introduced to facilitate the presentation of the results in nondimensional form. In fact, the article inadvertently and incorrectly introduces δ, which is the viscous layer thickness in the definition of a. The proper quantity that should have appeared in Eq. (13) and the subsequent definition of the Reynolds number is y_∞, which is the normal distance far upstream. Corrections were indeed sent to the *AIAA Journal*, but unfortunately they appear to have arrived too late to be included in the final published paper.

Received Aug. 9, 1995; accepted for publication Aug. 10, 1995.

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